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In The Subject

**MATHEMATICS**

Submitted by

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## PROJECT COMPLETION REPORT

This is to certify that the Minor Research Project entitled *RAM Finite Hyperbolic Transforms* is a genuine research work done by **Dr. Raju Ambadas Muneshwar** at P. G. Department of Mathematics, N. E. S., Science College, Nanded, (MH), India - 431605, for the fulfillment of Minor Research Project in Mathematics under the Faculty of Science as per the requirements of University Grants Commission, Delhi.

This work has not been previously submitted in whole or in part, for the award of any degree of this or any other University.

Date:

Place: Nanded

Dr. L. P. Shinde

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## DECLARATION

I hereby declare that the Minor Research Project (MRP) entitled *On RAM Finite Hyperbolic Transforms* Sanctioned by University Grants Commission is original work carried out by me in the department of Mathematics, N. E. S. Science College, Nanded (Maharashtra), India.

Date:

Place: Nanded

Dr. Raju Ambadas Muneshwar  
Principal Investigator

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I extend my thanks to all, who have helped me during the course of my research work. It is possible that some omissions, occasional errors and lapses in accuracy of interpretation may have escaped my notice. I accept full and exclusive responsibility for the same.

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Principal Investigator

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## ABSTRACT

The minor research project work is arranged in seven chapters as follows.

The first chapter contains introduction of the different integral transforms. The second chapter contains review of literature in which some basic definitions and some existing results have been given.

In the third chapter, a ***RAM Finite Sine Hyperbolic Transforms***, denoted by  $R_{sh}$  & ***RAM Finite Cosine Hyperbolic Transforms***, denoted by  $R_{ch}$  are introduced. The  $R_{sh}$  &  $R_{ch}$  of some standard functions such as exponential function, trigonometric function, polynomial function, power function etc. are discussed in this chapter. Moreover, we also derive necessary and sufficient condition for existence of RAM Finite Sine Hyperbolic Transform and RAM Finite Cosine Hyperbolic Transform.

In the fourth chapter, we will discussed Some properties such as Linearity Property, Scalar Multiplication, Scaling Property, Shifting Property etc, of ***RAM Finite Sine Hyperbolic Transforms***, & ***RAM Finite Cosine Hyperbolic Transforms***. Some other properties of these RAM Hyperbolic Transforms are also discussed. Moreover, we will also discussed the applications of these properties.

In the fifth chapter, we will discussed Derivatives of RAM Finite Sine Hyperbolic Transform, Integration of RAM Finite Sine Hyperbolic Transform, RAM Finite Sine Hyperbolic Transform of Integration and RAM Finite Sine Hyperbolic Transform of Derivatives. Furthermore, we also discussed the applications of these properties.

In the sixth chapter, we have discussed the Derivatives of the RAM Finite Cosine Hyperbolic Transform, the Integration of the RAM Finite Cosine Hyperbolic Transform, the RAM Finite Cosine Hyperbolic Transform of Integration, and the RAM Finite Cosine Hyperbolic Transform of Derivatives. These topics have been examined in detail to establish their fundamental properties and mathematical relationships.

In the seventh chapter, we will discuss the RAM Finite Hyperbolic Transforms in greater depth. The RAM Finite Hyperbolic Transform, defined over a finite

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interval, provides an efficient framework for analyzing and representing finite signals and functions. It serves as an important mathematical tool for simplifying complex analytical problems and for developing further theoretical results related to finite transforms.

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# Chapter 1

## Introduction

In this chapter, we will discuss history and development of different integral transformations, including the Laplace and Fourier transforms. The modern form of the Laplace transform:

$$F(s) = \int_0^{\infty} e^{-st} f(t), dt$$

was developed in the 19th and early 20th centuries, with contributions from several mathematicians, including: Oliver Heaviside (1850-1925): An English electrical engineer and mathematician, Heaviside used the Laplace transform to solve differential equations in electrical circuit analysis. The history of integral transformations is marked by the contributions of many mathematicians who developed and refined these tools over centuries. From the early work of Euler and Fourier to the formalization by Laplace and the advancements in the 20th century, integral transforms have become indispensable in both theoretical and applied mathematics. Their development reflects the deepening understanding of mathematical analysis and its profound impact on science and engineering.

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## Review of the Literature

### 2.1 Introduction

Integral transforms have been widely used for nearly two centuries to solve problems in applied mathematics, mathematical physics, and engineering science. The origins of these transforms, including the Laplace and Fourier transforms, trace back to the pioneering work of P. S. Laplace (1749–1827) on probability theory in the 1780s and Joseph Fourier (1768–1830) in his monumental treatise *La Théorie Analytique de la Chaleur* (1822). Laplace’s *La Théorie Analytique des Probabilités* introduced fundamental results of what is now known as the Laplace transform, a powerful tool for solving linear differential and integral equations. Fourier’s work, on the other hand, developed the modern theory of heat conduction and established the Fourier Integral Theorem and Fourier series, enabling the representation of arbitrary functions on finite intervals in terms of trigonometric series.

### 2.2 Some Useful Definitions

In this section, we will recall some important definitions, results and notations used in the thesis of different integral transforms .

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# RAM Finite Hyperbolic Transforms

## 3.1 Introduction

Finite Laplace transform motivate us to define RAM Finite Sine Hyperbolic transform and RAM Finite Cosine Hyperbolic transform in  $0 \leq t \leq T$  in order to extend the power and usefulness of usual Laplace transform in  $0 \leq t < \infty$ .

## 3.2 RAM Finite Hyperbolic Transforms

**Definition 1.1** Let  $p \in C$  and  $T$  be a finite number which may be positive or negative and  $f(t)$  is a continuous ( or an almost piecewise continuous ) function defined over the interval  $(0,T)$ . Then RAM Finite Sine Hyperbolic transform of  $f(t)$  is denoted by  $R_{sh}(f(t)) = F_S(p, T)$ , and defined by

$$R_{sh}(f(t)) = F_S(p, T) = \int_0^T \sinh(pt)f(t)dt,$$

where  $\sinh(pt)$  is a kernel of  $R_{sh}$ .

Here  $R_{sh}$  is called RAM Finite Sine Hyperbolic transformation operator.

**Definition 1.2** Let  $p \in C$  and  $T$  be a finite number which may be positive or

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negative and  $f(t)$  is a continuous ( or an almost piecewise continuous ) function defined over the interval  $(0,T)$ . Then RAM Finite Cosine Hyperbolic transform of  $f(t)$  is denoted by  $R_{ch}(f(t)) = F_C(p, T)$ , and defined by

$$R_{ch}(f(t)) = F_C(p, T) = \int_0^T \cosh(pt)f(t)dt,$$

where  $\cosh(pt)$  is a kernel of  $R_{ch}$ .

Here  $R_{ch}$  is called RAM Finite Cosine Hyperbolic transformation operator.

**Note :** That  $\sinh t$ ,  $\cosh t$  are bounded for any bounded interval  $(-T_1, T_2)$ .

### 3.3 Existence of $R_{sh}$ and $R_{ch}$

**Theorem 3.3.1.** *If  $f(t)$  is a piecewise continuous and absolutely integrable function on  $(0, T)$ , then  $R_{sh}(f(t))$  exists.*

**Theorem 3.3.2.** *If  $f(t)$  is a piecewise continuous and absolutely integrable function on  $(0, T)$ , then  $R_{ch}(f(t))$  exists.*

**Theorem 3.3.3.** *If  $f(t)$  is a piecewise continuous and bounded function on  $(0, T)$ , then  $R_{sh}(f(t))$  exists.*

**Theorem 3.3.4.** *If  $f(t)$  is a piecewise continuous and bounded function on  $(0, T)$ , then  $R_{ch}(f(t))$  exists.*

### 3.4 RAM Finite Sine Hyperbolic transform of some standard functions.

1.  $R_{sh}(1) = \frac{\cosh(pt)}{p}$ .

$$2. R_{sh}(t) = \frac{T \cosh(pT)}{p} - \frac{\sinh(pT)}{p^2}.$$

$$3. R_{sh}(t^2) = \frac{T^2 \cosh(pT)}{p} - \frac{2T \sinh(pT)}{p^2} + \frac{(2 \cosh(pT) - 2)}{p^3}.$$

$$4. R_{sh}(t^k) = \begin{cases} \frac{T^k \cosh(pT)}{p} - \frac{kT^{k-1} \sinh(pT)}{p^2} + \dots + \frac{k!(-1)^k [\cosh(pT) - 1]}{p^k}, & \text{if } k \text{ is even} \\ \frac{T^k \cosh(pT)}{p} - \frac{kT^{k-1} \sinh(pT)}{p^2} + \dots + \frac{k!(-1)^k \sinh(pT)}{p^k}, & \text{if } k \text{ is odd.} \end{cases}$$

$$5. R_{sh}(\sin(at)) = \left( \frac{-a}{p^2 + a^2} \right) \sinh(pT) \cos(aT) + \left( \frac{p}{p^2 + a^2} \right) \cosh(pT) \sin(aT).$$

$$6. R_{sh}(\cos(at)) = \left( \frac{a}{p^2 + a^2} \right) \sinh(pT) \cdot \sin(aT) + \left( \frac{p}{p^2 + a^2} \right) [\cosh(pT) \cdot \cos(aT) - 1].$$

$$7. R_{sh}(e^{at}) = \left( \frac{-a}{p^2 - a^2} \right) \sinh(pT) \cdot e^{aT} + \left( \frac{p}{p^2 - a^2} \right) [\cosh(pT) \cdot e^{aT} - 1],$$

provided  $p^2 \neq a^2$

$$8. R_{sh}(e^{-at}) = \left( \frac{a}{p^2 - a^2} \right) \sinh(pT) \cdot e^{-aT} + \left( \frac{-p}{p^2 - a^2} \right) [1 - \cosh(pT) \cdot e^{-aT}],$$

provided  $p^2 \neq a^2$ .

### 3.5 RAM Finite Cosine Hyperbolic transform of some standard functions.

$$1. R_{ch}(1) = \frac{\sinh(pT)}{p}.$$

$$2. R_{ch}(t) = \frac{T \cosh(pT)}{p} - \frac{\cosh(pT) - 1}{p^2}.$$

$$3. R_{ch}(t^2) = \frac{T^2 \cdot \sinh(pT)}{p} - \frac{2T \cdot \cosh(pT)}{p^2} + \frac{2 \cdot \sinh(pT)}{p^3}.$$

$$4. R_{ch}(t^k) = \begin{cases} \frac{T^k \sinh(pT)}{p} - \frac{kT^{k-1} \cosh(pT)}{p^2} + \dots + \frac{k!(-1)^k \sinh(pT)}{p^k} & \text{if } k \text{ is even} \\ \frac{T^k \cdot \sinh(pT)}{p} - \frac{kT^{k-1} \cosh(pT)}{p^2} + \dots + \frac{k!(-1)^k [\cosh(pT)-1]}{p^k}, & \text{if } k \text{ is odd.} \end{cases}$$

$$5. R_{ch}(\sin(at)) = \left(\frac{a}{p^2+a^2}\right) [1 - \cosh(pT) \cos(aT)] + \left(\frac{p}{p^2+a^2}\right) \sinh(pT) \sin(aT).$$

$$6. R_{ch}(\cos(at)) = \left(\frac{a}{p^2+a^2}\right) \cosh(pT) \sin(aT) + \left(\frac{p}{p^2+a^2}\right) \sinh(pT) \cos(aT).$$

$$7. R_{ch}(e^{at}) = \left(\frac{a}{p^2-a^2}\right) [\cosh(pT)e^{aT} - 1] + \left(\frac{p}{p^2-a^2}\right) \sinh(pT)e^{aT},$$

provided  $p^2 \neq a^2$ .

$$8. R_{ch}(e^{-at}) = \left(\frac{a}{p^2-a^2}\right) \cosh(pT)e^{-aT} + \left(\frac{-p}{p^2-a^2}\right) [1 - \sinh(pT)e^{-aT}],$$

provided  $p^2 \neq a^2$ .

# Chapter 4

## Some properties of RAM Finite

## Hyperbolic transform

### 4.1 Introduction

In this section , the concept of RAM Finite Hyperbolic Transforms is introduced.

### 4.2 Some properties of RAM Finite Sine Hyperbolic transform

1. **Linearity:**  $R_{sh}(f_1(t) + f_2(t)) = R_{sh}(f_1(t)) + R_{sh}(f_2(t))$ .

2. **Scalar Multiplication:** If  $c$  be any constant, then  $R_{sh}(cf(t)) = cR_{sh}(f(t))$ .

3. **Scaling:** If  $R_{sh}(f(t)) = F_S(p, T)$  then  $R_{sh}(f(at)) = \frac{F_S(\frac{p}{a}, aT)}{a}$

#### 4. Shifting Properties .

(I). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\cosh(at).f(t)) + R_{ch}(\sinh(at).f(t)) = F_S((p + a), T)$$

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(II). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\cosh(at).f(t)) - R_{ch}(\sinh(at).f(t)) = F_S((p - a), T)$$

(III). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\cosh(at).f(t)) = \frac{F_S((p + a), T) + F_S((p - a), T)}{2}$$

(IV). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\sinh(at).f(t)) = \frac{F_C((p + a), T) - F_C((p - a), T)}{2}$$

(V). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(e^{-at}.f(t)) = \frac{F_S((p + a), T) + F_S((p - a), T) + F_C((p - a), T) - F_C((p + a), T)}{2}$$

(VI). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(e^{at}.f(t)) = \frac{F_S((p + a), T) + F_S((p - a), T) - F_C((p - a), T) + F_C((p + a), T)}{2}$$

(VII). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\cos(at).f(t)) = \frac{F_S((p + ia), T) + F_S((p - ia), T)}{2}$$

(VIII). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\sin(at).f(t)) = \frac{F_C((p + ia), T) - F_C((p - ia), T)}{2i}$$

(IX). Suppose  $f(t) = 0$  for  $t < 0$ . If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(f(t - a)) = \sinh(pa)F_C(p, (T - a)) + \cosh(pa)F_S(p, (T - a))$$

### 4.3 Some properties of RAM Finite Cosine Hyperbolic transform

1. **Linearity:**  $R_{ch}(f_1(t) + f_2(t)) = R_{ch}(f_1(t)) + R_{ch}(f_2(t))$

**2. Scalar Multiplication:** If  $c$  is any constant, then  $R_{ch}(c.f(t)) = c.R_{ch}(f(t))$ .

**3. Scaling:** If  $R_{ch}(f(t)) = F_C(p, T)$ , then  $R_{ch}(f(at)) = \frac{F_C(\frac{p}{a}, aT)}{a}$

#### 4. Shifting Properties :

(I). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(\cosh(at).f(t)) - R_{sh}(\sinh(at).f(t)) = F_C((p - a), T)$$

(II). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{sh}(\sinh(at).f(t)) + R_{ch}(\cosh(at).f(t)) = F_C((p + a), T)$$

(III). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(\cosh(at).f(t)) = \frac{F_C((p + a), T) + F_C((p - a), T)}{2}$$

(IV). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(\sinh(at).f(t)) = \frac{-F_S((p - a), T) + F_S((p + a), T)}{2}$$

(V). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(e^{-at}.f(t)) = \frac{F_C((p - a), T) + F_C((p + a), T) + F_S((p - a), T) - F_S((p + a), T)}{2}$$

(VI). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(e^{at}.f(t)) = \frac{F_C((p - a), T) + F_C((p + a), T) - F_S((p - a), T) + F_S((p + a), T)}{2}$$

(VII). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(\cos(at).f(t)) = \frac{F_C((p + ia), T) + F_C((p - ia), T)}{2}$$

(VIII). If  $R_{sh}(f(t)) = F_S(p, T)$  and  $R_{ch}(f(t)) = F_C(p, T)$ , then

$$R_{ch}(\sin(at).f(t)) = \frac{-F_S((p - ia), T) + F_S((p + ia), T)}{2i}$$

## 4.4 Examples

In this section we will find RAM Finite Sine Hyperbolic Transforms of some standard functions.

# Chapter 5

## RAM Finite Sine Hyperbolic Transforms of Derivatives and Integration of $f(t)$

### 5.1 Introduction and RAM Finite Hyperbolic Transforms

In this section , we recall the concept of RAM Finite Hyperbolic Transforms.

### 5.2 Derivatives of RAM Finite Sine Hyperbolic Transform

**Theorem 5.2.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d}{dp} (S_H(f(t)) = C_H (tf(t))),$$

**Theorem 5.2.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d^2}{dp^2} (S_H(f(t)) = S_H (t^2 f(t))),$$

**Theorem 5.2.3.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d^3}{dt^3} (S_H(f(t)) = C_H (t^3 f(t))),$$

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**Theorem 5.2.4.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d^k}{dt^k} (S_H(f(t)) = C_H(t^k f(t))), \text{ If } k \text{ is odd}$$

$$\frac{d^k}{dt^k} (S_H(f(t)) = S_H(t^k f(t))), \text{ If } k \text{ is even}$$

### 5.3 Integration of RAM Finite Sine Hyperbolic Transform

**Theorem 5.3.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\int_p^T (S_H(f(t)) = G_C(T, T) - G_C(p, T))$$

where  $G(t) = \frac{f(t)}{t}$

**Theorem 5.3.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\int_p^T \int_p^T S_H(f(t)) = TG_C(T, T) - pG_C(T, T) - H_S(T, T) + H_S(p, T)$$

where  $G(t) = \frac{f(t)}{t}$  and  $H(t) = \frac{f(t)}{t^2}$

### 5.4 RAM Finite Sine Hyperbolic Transform of Integration

**Theorem 5.4.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H \left( \int_0^t f(u) du \right) = \frac{\cosh(pT)g(T) - g(0) - C_H(f(t))}{p}$$

**Theorem 5.4.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H \left( \int_0^t \int_0^t f(u) du du \right) = \frac{-\sinh(pT)f'(T) + pf(T) \cosh(pT) + pf(0) + S_H(f(t))}{p^2}$$

**Theorem 5.4.3.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H \left( \int_0^t \int_0^t \int_0^t f(u) du du du \right) = \frac{\sinh(pT)g''(T) - 0g''(0) - pg'(T) \cosh(pT) + pg'(0) + p^2g(T) \sin(pT)}{p^3}$$

**Theorem 5.4.4.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H (f^k(t)) = \sin p T f'(T) - 0f'(0) - pf(T) \cosh(pT) + pf(0) + p^2 F_S(p, T)$$

## 5.5 RAM Finite Sine Hyperbolic Transform of Derivatives

**Theorem 5.5.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H (f^{(1)}(t)) = \sinh(pT)f(T) - pC_H(f(t))$$

**Theorem 5.5.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H (f^2(t)) = \sinh(pT)f^{(1)}(T) - 0f^{(1)}(0) - pf(T) \cosh(pT) + pf(0) + p^2 S_H(f(t))$$

**Theorem 5.5.3.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H (f^{(3)}(t)) = \sinh(pT)f^{(2)}(T) - 0f^{(2)}(0) - pf^{(1)}(T) \cosh(pT) + pf^{(1)}(0) + p^2 f(T) \sinh(pT) - p^2 f(T)0 - p^3 C_H(f(t))$$

**Theorem 5.5.4.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H (f^k(t)) = \sinh(pT)f^{(k-1)}(T) - 0f^{(k-1)}(0) - pf^{(k-2)}(T) \cosh(pT) + pf^{(k-2)}(0) + p^2 S_H(f^{(k-2)})$$

# Chapter 6

## RAM Finite Cosine Hyperbolic

## Transforms of Derivatives and Integration

## of $f(t)$

### 6.1 Introduction and RAM Finite Hyperbolic Transforms

In this section, we recall the concept of RAM Finite Hyperbolic Transforms.

### 6.2 Derivatives of RAM Finite Cosine Hyperbolic Transform

**Theorem 6.2.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d}{dt} (C_H(f(t))) = S_H(tf(t))$$

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**Theorem 6.2.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d^2}{dt^2} (C_H(f(t)) = C_H(t^2 f(t)))$$

**Theorem 6.2.3.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d^3}{dt^3} (C_H(f(t)) = S_H(t^3 f(t)))$$

**Theorem 6.2.4.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\frac{d^k}{dt^k} (S_H(f(t)) = C_H(t^k f(t))), \text{ if } k \text{ is odd}$$

$$\frac{d^k}{dt^k} (S_H(f(t)) = S_H(t^k f(t))), \text{ if } k \text{ is even}$$

## 6.3 Integration of RAM Finite Cosine Hyperbolic Transform

**Theorem 6.3.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\int_p^T (C_H(f(t)) = G_S(T, T) - G_S(p, T));$$

where  $G(t) = \frac{f(t)}{t}$

**Theorem 6.3.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$\int_p^T \int_p^T S_H(f(t)) = TG_S(T, T) - pG_S(T, T) - H_C(T, T) + H_C(p, T)$$

where  $G(t) = \frac{f(t)}{t}$  and  $H(t) = \frac{f(t)}{t^2}$

## 6.4 RAM Finite Cosine Hyperbolic Transform of Integration

**Theorem 6.4.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$C_H \left( \int_0^t f(u) du \right) = \frac{\sin(pT)g(T) - S_H(f(t))}{p}$$

**Theorem 6.4.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$C_H \left( \int_0^t \int_0^t f(u) du du \right) = \frac{\cosh(pT)g'(T) - g'(0) - pg(T) \sinh(pT) - C_H(f(t))}{p^2}$$

**Theorem 6.4.3.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$C_H \left( \int_0^t \int_0^t f(u) du du \right) = \frac{\cosh(pT)g'(T) - g'(0) - pg(T) \sinh(pT) - C_H(f(t))}{p^2}$$

**Theorem 6.4.4.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$S_H(f^k(t)) = \sin(pT)f'(T) - 0f'(0) - pf(T) \cosh(pT) + pf(0) + p^2 F_S(p, T)$$

## 6.5 RAM Finite Cosine Hyperbolic Transform of Derivatives

**Theorem 6.5.1.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$C_H(f^{(1)}(t)) = \cosh(pT)f(T) - f(0) - pS_H(f(t))$$

**Theorem 6.5.2.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$C_H(f^2(t)) = \cosh(pT)f^1(T) - f^1(0) - pf(T) \sinh(pT) + pf(0)0 + p^2 C_H(f(t))$$

**Theorem 6.5.3.** *If  $R_{ch}(f(t)) = F_C(p, T)$ , then*

$$C_H(f^k(t)) = \cosh(pT)f^{k-1}(T) - f^{k-1}(0) - pf^{(k-2)}(T) \sinh(pT) + 0pf^{(k-2)}(0) + p^2 C_H(f^{(k-2)})$$

# Applications of of RAM Finite Hyperbolic Transforms

In this chapter, we will discuss some of the applications of RAM Finite Hyperbolic Transforms

## 7.1 RAM Finite Hyperbolic Transforms

**Definition 7.1** Let  $p \in C$  and  $T$  be a finite number which may be positive or negative and  $f(t)$  is a continuous ( or an almost piecewise continuous ) function defined over the interval  $(0,T)$ . Then RAM Finite Sine Hyperbolic transform of  $f(t)$  is denoted by  $R_{sh}(f(t)) = F_S(p, T)$ , and defined by

$$R_{sh}(f(t)) = F_S(p, T) = \int_0^T \sinh(pt)f(t)dt,$$

where  $\sinh(pt)$  is a kernel of  $R_{sh}$ .

Here  $R_{sh}$  is called RAM Finite Sine Hyperbolic transformation operator.

**Definition 7.2** Let  $p \in C$  and  $T$  be a finite number which may be positive or negative and  $f(t)$  is a continuous ( or an almost piecewise continuous ) function defined over the interval  $(0,T)$ . Then RAM Finite Cosine Hyperbolic transform of

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$f(t)$  is denoted by  $R_{ch}(f(t)) = F_C(p, T)$ , and defined by

$$R_{ch}(f(t)) = F_C(p, T) = \int_0^T \cosh(pt) f(t) dt,$$

where  $\cosh(pt)$  is a kernel of  $R_{ch}$ .

Here  $R_{ch}$  is called RAM Finite Cosine Hyperbolic transformation operator.

**Note :** That  $\sinh t$ ,  $\cosh t$  are bounded for any bounded interval  $(-T_1, T_2)$ .

## 7.2 Applications of of RAM Finite Hyperbolic Transforms

The RAM Finite Hyperbolic Transforms is applied over a finite interval. It has several practical applications across various fields. Here are some key applications:

### 1. Signal Processing:

**Time-Limited Signals:** Many signals in practical applications are of finite duration. The RAM Finite Hyperbolic Transforms is particularly useful in analyzing such time-limited signals.

**Filter Design:** It helps in designing filters for signals that exist over a finite interval, ensuring accurate analysis and manipulation of these signals.

### 2. Control Systems:

**2.i Transient Analysis:** In control systems, analyzing the transient response is crucial. The RAM Finite Hyperbolic Transforms can be used to study the behavior of systems within a finite time period.

**2.ii System Identification:** It helps in identifying system parameters from finite-duration data, which is important in real-world scenarios where infinite-duration data is not available.

### 3. Vibration Analysis:

**3.i Structural Health Monitoring:** For structures subjected to vibrations for a finite period, the RAM Finite Hyperbolic Transforms aids in analyzing and identifying potential issues.

**3.ii Modal Analysis:** It helps in determining the modes of vibration within a finite

time frame, which is essential for structural analysis and design.

#### 4. Financial Mathematics:

**4.i Option Pricing:** The RAM Finite Hyperbolic Transforms can be applied to price options and other financial derivatives over a finite time horizon.

**4.ii Risk Management:** It is useful in modeling and managing risks over finite investment periods.

#### 5. Electromagnetic Theory:

**5.i Wave Propagation:** In electromagnetic wave propagation problems, the RAM Finite Hyperbolic Transforms helps in analyzing waves that are confined within a finite spatial or temporal domain.

**5.ii Antenna Design:** It aids in the design and analysis of antennas where the signals are of finite duration.

#### 6. Biomedical Engineering:

**6.i Signal Analysis:** Used in analyzing biomedical signals (e.g., ECG, EEG) which are often captured over finite periods.

**6.ii Imaging Techniques:** Helps in the reconstruction and processing of images in medical imaging systems where data collection is over a finite duration.

#### 7. Communication Systems:

**7.i Finite Duration Transmission:** In communication systems, the finite Laplace transform is applied to analyze signals transmitted over finite intervals, ensuring effective modulation and demodulation techniques.

#### Mathematical Formulation:

Then RAM Finite Sine Hyperbolic transform of  $f(t)$  is denoted by  $R_{sh}(f(t)) = F_S(p, T)$ , and defined by

$$R_{sh}(f(t)) = F_S(p, T) = \int_0^T \sinh(pt)f(t)dt,$$

where  $\sinh(pt)$  is a kernel of  $R_{sh}$ .

Then RAM Finite Cosine Hyperbolic transform of  $f(t)$  is denoted by  $R_{ch}(f(t)) = F_C(p, T)$ , and defined by

$$R_{ch}(f(t)) = F_C(p, T) = \int_0^T \cosh(pt)f(t)dt,$$

where  $\cosh(pt)$  is a kernel of  $R_{ch}$ .

**Example 1.** Suppose  $f(t) = e^{-at}$ , for  $t \in [0, T]$ .

The RAM Finite Cosine Hyperbolic transform of  $f(t) = e^{-at}$  is:  $R_{ch}(e^{-at}) = \left(\frac{a}{p^2 - a^2}\right) [\cosh(pT)e^{aT} - 1] + \left(\frac{p}{p^2 - a^2}\right) \sinh(pT)e^{aT}$ , provided  $p^2 \neq a^2$ .

This transform can be used in various applications, as outlined above, to analyze and interpret signals and systems confined within a finite domain. In summary, the RAM Finite Hyperbolic transform is a versatile tool that extends the utility of the traditional RAM Finite Hyperbolic transform to practical scenarios involving finite time intervals, making it invaluable across multiple disciplines.

## Future Work

The present study has developed and analyzed various properties of the RAM Finite Sine Hyperbolic Transform (RSHT) and RAM Finite Cosine Hyperbolic Transform (RCHT) and demonstrated their effectiveness in solving mathematical problems. However, this area still offers several promising directions for future research and exploration:

### 8.1 Future Work

The RAM Finite Hyperbolic Transforms is applied over a finite interval. It has several practical applications across various fields. Here are some key applications:

**1. Development of Transform-Based Theorems:**

Further research can focus on establishing operational theorems, duality relationships, and energy-preserving identities to enrich the theoretical foundation of these transforms.

**2. Development of Transform-Based Convolution Theorems:**

Further research can focus on establishing Transform-Based Convolution Theorems.

**3. Extension to Higher Dimensions:**

The current study focuses on single-variable functions. Future work may extend these transforms to two-dimensional and multidimensional cases, enabling applications in image processing, fluid dynamics, and electromagnetics.

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**4. Development of Inverse Transform Formulas:**

Deriving and proving inverse RAM Finite Hyperbolic Transform formulas will be an essential step for reconstructing original functions and for practical applications in data recovery and signal analysis.

**5. Numerical and Computational Algorithms:**

Designing efficient numerical algorithms and computer-based methods for evaluating RAM Finite Hyperbolic Transforms will enhance their usability in scientific computing and simulation.

**6. Application to Differential and Integral Equations:**

The transforms can be further applied to solve more complex partial differential equations (PDEs), integro-differential equations, and boundary value problems arising in heat conduction, vibration analysis, and diffusion processes..

**7. Comparative Study with Other Transforms:**

Comparative analysis between the RAM Finite Hyperbolic Transforms and classical transforms such as Laplace, Fourier, Mellin, and Hankel transforms could yield deeper insights and reveal conditions under which each transform performs optimally

**8. Generalization and Hybrid Forms:**

Developing generalized or hybrid forms of the RAM Finite Hyperbolic Transform by combining it with other transform techniques could lead to new tools for tackling complex analytical and computational problems.

**9. Applications in Engineering and Physical Sciences:**

Future studies may explore the use of these transforms in signal and image processing, control theory, mechanical vibrations, thermal analysis, and electromagnetic field modeling within finite or bounded regions.

**10. Software and Visualization Tools:**

Implementing the RAM Finite Hyperbolic Transforms into software packages (e.g., MATLAB, Python, or Mathematica) and developing graphical visualization tools will make these transforms more accessible to researchers and engineers

**11. Theoretical Analysis of Convergence and Stability:**

A rigorous study on the convergence, stability, and error estimation of the RAM Finite Hyperbolic Transforms will help in establishing their reliability and precision

in both analytical and numerical computations

**12. Exploration of Fractional and Discrete Versions:**

Extending the concept to fractional-order and discrete versions of the RAM Finite Hyperbolic Transforms could open new avenues for research in fractional calculus, discrete systems, and digital signal processing

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## List of Publications

### List of Publications

1. Some Results on Derivatives and Integration of the RAM Finite Sine Hyperbolic Transform, Ajanata Journal of Mathematics, Vol. No. 5, Issue No. II, ISSN: 2249-0604, pp. 52-59, Jul - Sept 2015, (UGC Approved Journal)
  2. Shifting Properties Of Finite Cosine Hyperbolic Transforms, International Journal of Research in Science And Technology, (IJRST), Vol. No. 5, Issue No. II, ISSN: 2249-0604, pp. 52-59, Apr-Jun 2015, (UGC Approved Journal)
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  4. On RAM Finite Hyperbolic Transforms, IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 10, Issue 6 Ver. IV PP 63-70, (Nov - Dec. 2014), (UGC Approved Journal).
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